Patterns and Linear Functions

Section 4-2
Goals

Goal

• To identify and represent patterns that describe linear functions.
Vocabulary

• Dependent Variable
• Independent Variable
• Input
• Output
• Function
• Linear Function
Definition

- **Dependent Variable** – A variable whose value depends on some other value.
  - Generally, $y$ is used for the dependent variable.

- **Independent Variable** – A variable that doesn’t depend on any other value.
  - Generally, $x$ is used for the independent variable.

- The value of the **dependent variable** depends on the value of the **independent variable**.
Independent and Dependent Variables

On a graph;
the **independent** variable is on the **horizontal** or x-axis.
the **dependent** variable is on the **vertical** or y-axis.
Example:

Identify the independent and dependent variables in the situation.

A painter must measure a room before deciding how much paint to buy.

The amount of paint depends on the measurement of a room.

Dependent: amount of paint
Independent: measurement of the room
Example:

Identify the independent and dependent variables in the situation.

The height of a candle decrease $d$ centimeters for every hour it burns.

The height of a candle depends on the number of hours it burns.

Dependent: height of candle
Independent: time
Example:

Identify the independent and dependent variables in the situation.

A veterinarian must weight an animal before determining the amount of medication.

The amount of medication depends on the weight of an animal.

Dependent: amount of medication
Independent: weight of animal
A company charges $10 per hour to rent a jackhammer.

Identify the independent and dependent variable in the situation.

A company charges $10 per hour to rent a jackhammer.

The **cost to rent a jackhammer** depends on the **length of time it is rented**.

Dependent variable: **cost**
Independent variable: **time**
Your Turn:

Identify the independent and dependent variable in the situation.

Camryn buys $p$ pounds of apples at $0.99$ per pound.

The cost of apples depends on the number of pounds bought.

Dependent variable: cost
Independent variable: pounds
Example: Representing a Geometric Relationship

In the diagram below, what is the relationship between the number of rectangles and the perimeter of the figure they form? Represent this relationship using a table, words, an equation, and a graph.
Example: Representing a Geometric Relationship

Step 1

Make a table. Use the number of rectangles as the independent variable ($x$) and the perimeter as the dependent variable ($y$).

Perimeter = 2(length) + 2(width)

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
</tr>
</tbody>
</table>
Example: Representing a Geometric Relationship

Step 2

Look for a pattern in the table. How did you calculate the perimeter \( (y) \), given the number of rectangles \( (x) \)? Then describe the pattern in words.

Perimeter = 2(length) + 2(width)

<table>
<thead>
<tr>
<th>Number of Rectangles, ( x )</th>
<th>Perimeter, ( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2(1) + 2(6) = 14</td>
</tr>
<tr>
<td>2</td>
<td>2(2) + 2(6) = 16</td>
</tr>
<tr>
<td>3</td>
<td>2(3) + 2(6) = 18</td>
</tr>
<tr>
<td>4</td>
<td>2(4) + 2(6) = 20</td>
</tr>
</tbody>
</table>

**Words:** Multiply the number of rectangles in each figure by 2 to get the total length of the top and bottom sides of the combined figure. Then add 2(6), or 12, for the total length of the left and right sides of the combined figure to get the entire perimeter.
Example: Representing a Geometric Relationship

Step 3

From the pattern in the table write an equation to represent the relationship between $x$ and $y$.

<table>
<thead>
<tr>
<th>Number of Rectangles, $x$</th>
<th>Perimeter, $y$</th>
<th>Equation: $y = 2x + 12$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2(1) + 2(6) = 14$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$2(2) + 2(6) = 16$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$2(3) + 2(6) = 18$</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$2(4) + 2(6) = 20$</td>
<td></td>
</tr>
</tbody>
</table>
Example: Representing a Geometric Relationship

Step 4

Use the table to make a graph.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>Ordered Pair $(x, y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>14</td>
<td>(1, 14)</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td>(2, 16)</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
<td>(3, 18)</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>(4, 20)</td>
</tr>
</tbody>
</table>

With a graph, you can see a pattern formed by the relationship between the number of rectangles and the perimeter of the figure.
Your Turn:

In the diagram below, what is the relationship between the number of triangles and the perimeter of the figure they form? Represent the relationship using (1) a table, (2) words, (3) an equation, and (4) a graph.

1 triangle | 2 triangles | 3 triangles | 4 triangles
---|---|---|---
3 | 3 | 3 | 3
4 | 4 | 4 | 4
1 | 2 | 3 | 4
Answer:

1) Multiply the number of triangles by 4 and add 6.

<table>
<thead>
<tr>
<th>Number of Triangles</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perimeter</td>
<td>10</td>
<td>14</td>
<td>18</td>
<td>22</td>
</tr>
</tbody>
</table>

3) \( y = 4x + 6 \)
Definition

- **Input** – Values of the independent variable.
  - $x$ – values
  - The **input** is the value substituted into an equation.
- **Output** – Values of the dependent variable.
  - $y$ – values.
  - The **output** is the result of that substitution in an equation.
Function

• In the last 2 problems you can describe the relationship by saying that the perimeter (dependent variable – $y$ value) is a function of the number of figures (independent variable – $x$ value).

• A function is a relationship that pairs each input value with exactly one output value.
Function

You can think of a function as an input-output machine.

function $y = 5x$
There are several different ways to describe the variables of a function.

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Dependent Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$-values</td>
<td>$y$-values</td>
</tr>
<tr>
<td>Input</td>
<td>Output</td>
</tr>
<tr>
<td>Domain</td>
<td>Range</td>
</tr>
<tr>
<td>$x$</td>
<td>$f(x)$</td>
</tr>
</tbody>
</table>
A **function** is a set of ordered pairs \((x, y)\) so that each \(x\)-value corresponds to exactly one \(y\)-value.

Some functions can be described by a rule written in words, such as “double a number and then add nine to the result,” or by an equation with two variables. One variable \((x)\) represents the *input*, and the other variable \((y)\) represents the *output*. 

\[
y = 2x + 9
\]
Linear Function

- Another method of representing a function is with a graph.

- A *linear function* is a function whose graph is a nonvertical line or part of a nonvertical line.
Example: Representing a Linear Function

A DVD buyers club charges a $20 membership fee and $15 per DVD purchased. The table below represents this situation.

<table>
<thead>
<tr>
<th>Number of DVDs purchased</th>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total cost ($)</td>
<td>y</td>
<td>20</td>
<td>35</td>
<td>50</td>
<td>65</td>
<td>80</td>
<td>95</td>
</tr>
</tbody>
</table>

Find the first differences for the total cost.

Since the data shows a **constant** difference the pattern is **linear**.

If a pattern is linear then its graph is a straight **line**.
The equation $y = 15x + 20$ represents this situation.

This is called a **linear** equation.

<table>
<thead>
<tr>
<th>Number of DVDs purchased</th>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total cost ($)</td>
<td>$y$</td>
<td>20</td>
<td>35</td>
<td>50</td>
<td>65</td>
<td>80</td>
<td>95</td>
</tr>
</tbody>
</table>
Your Turn: The costs associated with being a member of a CD Club are presented in the table below.

Find the first differences and write an equation to represent the data pattern.

<table>
<thead>
<tr>
<th>Number of CDs purchased</th>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total cost ($)</td>
<td>y</td>
<td>26</td>
<td>39</td>
<td>52</td>
<td>65</td>
<td>78</td>
<td>91</td>
</tr>
</tbody>
</table>

The club charges $13 per CD.
The cost for 0 CDs is $26.
Therefore, the club membership (initial cost) must be $26.

\[ y = 13x + 26 \]
Your Turn:

The table shows the costs associated with being a member of a DVD club that charges a membership fee. Write an equation to represent the pattern in the data.

<table>
<thead>
<tr>
<th>Number of DVDs purchased</th>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total cost ($)</td>
<td>$y$</td>
<td>21</td>
<td>31</td>
<td>41</td>
<td>51</td>
<td>61</td>
<td>71</td>
</tr>
</tbody>
</table>

$y = 10x + 21$
Your Turn:

The table shows the amount of water $y$ in a tank after $x$ minutes of being drained.

1. Is the relationship function?
2. Describe the relationship using words.
3. Write an equation for the relationship.

1. The relationship is a function.
2. The amount of water in gallons left in the tank is 440 minus 12 times the number of minutes.
3. $y = 440 – 12x$
Joke Time

• Why did the pilgrims' pants always fall down?
  • Because their belts were on their hats.

• What kind of birds flock together?
  • Vel-crows.

• What is the difference between a freshman and a cell phone.
  • You can put a cell phone on silent.