Goals

Goal

• To solve equations involving absolute value.
Vocabulary

- None
Absolute Value Equations

Recall that the absolute-value of a number is that number’s distance from zero on a number line. For example, \(|-5| = 5\) and \(|5| = 5\).

For any nonzero absolute value, there are exactly two numbers with that absolute value. For example, both 5 and \(-5\) have an absolute value of 5.

To write this statement using algebra, you would write \(|x| = 5\). This equation asks, “What values of \(x\) have an absolute value of 5?” The solutions are 5 and \(-5\). Notice this equation has two solutions.
## Absolute Value Equations

The equation $|x| = a$ asks, “What values of $x$ have an absolute value of $a$?” The solutions are $a$ and the opposite of $a$.

### Words

- The equation $|x| = a$ asks, “What values of $x$ have an absolute value of $a$?”
- The solutions are $a$ and the opposite of $a$.

### Numbers

- $|x| = 5$
- $x = 5$ or $x = -5$

### Graph

- The graph shows the absolute value $|x|$.
- The solutions are at $x = a$ and $x = -a$.

### Algebra

- $|x| = a$
- $x = a$ or $x = -a$
- $(a \geq 0)$
Absolute Value Equations

• To solve absolute-value equations, perform inverse operations to isolate the absolute-value expression on one side of the equation.
• Then you must consider two cases, one positive and the other negative.

Key Concept: Solving Absolute Value Equations

To solve an equation in the form $|A| = b$, where $A$ represents a variable expression and $b > 0$, solve $A = b$ and $A = -b$. 
Example: Solve Absolute Value Equations

Solve the equation.

\[ |x| = 12 \]

Think: What numbers are 12 units from 0?

Rewrite the equation as two cases, one equal to the positive and one equal to the negative.

Case 1

\[ x = 12 \]

Case 2

\[ x = -12 \]

The solution set is \{12, -12\}. 
To solve absolute value equations, you can use the fact that the expression inside the absolute value symbols can be either positive or negative.

You can solve some absolute-value equations using mental math. For instance, you learned that the equation $|x| = 8$ has two solutions: 8 and -8.

To solve absolute-value equations, you can use the fact that the expression inside the absolute value symbols can be either positive or negative.
Solving an Absolute-Value Equation

Solve \( |x-2| = 5 \)

**SOLUTION**

The expression \( x-2 \) can be equal to 5 or -5.

\( x-2 \) IS POSITIVE

\[
\begin{align*}
    x-2 &= 5 \\
    x &= 7
\end{align*}
\]

\( x-2 \) IS NEGATIVE

\[
\begin{align*}
    x-2 &= -5 \\
    x &= -3
\end{align*}
\]

The equation has two solutions: 7 and -3.

**CHECK**

\[
\begin{align*}
    |7 - 2| &= |5| = 5 \\
    |-3 - 2| &= |-5| = 5
\end{align*}
\]
Solving an Absolute-Value Equation

Solve $|2x-7|-5 = 4$

**SOLUTION**
Isolate the absolute value expression on one side of the equation.

$2x-7$ is POSITIVE

$|2x-7|-5 = 4$

$|2x-7| = 9$

$2x-7 = +9$

$2x = 16$

$x = 8$

$2x-7$ is NEGATIVE

$|2x-7|-5 = 4$

$|2x-7| = 9$

$2x-7 = -9$

$2x = -2$

$x = -1$

TWO SOLUTIONS $x = 8$ and $x = -1$
Solving Absolute Value Equations

The table summarizes the steps for solving absolute-value equations.

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<td>1. Use inverse operations to <strong>isolate the absolute-value</strong> expression.</td>
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<tr>
<td>2. <strong>Rewrite</strong> the resulting equation as two cases, one equal to the <strong>positive</strong> and one the <strong>negative</strong>, without absolute value notation.</td>
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<td>3. <strong>Solve</strong> the equation in each of the two cases.</td>
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Example: Solve Absolute Value Equations

Solve the equation.

\[3|x + 7| = 24\]

\[\frac{3|x + 7|}{3} = \frac{24}{3}\]

\[|x + 7| = 8\]

**Case 1**

\[x + 7 = 8\]

\[-7\]

\[x = 1\]

**Case 2**

\[x + 7 = -8\]

\[-7\]

\[x = -15\]

Since \(|x + 7|\) is multiplied by 3, divide both sides by 3 to undo the multiplication.

Think: What numbers are 8 units from 0?

Rewrite the equations as two cases, one equal to the positive and one equal to the negative. Since 7 is added to \(x\) subtract 7 from both sides of each equation.

The solution set is \(\{1, -15\}\).
Be sure to check both solutions when you solve an absolute-value equation.

\[
\frac{|x| = 4}{|−4| \quad 4 \quad 4 \checkmark} \quad \frac{|x| = 4}{|4| \quad 4 \quad 4 \checkmark}
\]
Your Turn:

Solve the equation.

\[ |x| - 3 = 4 \]

Isolate the absolute value expression. Since 3 is subtracted from \(|x|\), add 3 to both sides.

\[ |x| = 7 \]

Think: What numbers are 7 units from 0?

Rewrite the equation as two cases.

**Case 1**
\[ x = 7 \]

**Case 2**
\[ x = -7 \]

The solution set is \( \{7, -7\} \).
Your Turn:

Solve the equation.

$8 = |x - 2.5|$

**Case 1**

$8 = x - 2.5$

$+2.5 +2.5$

$10.5 = x$

**Case 2**

$-8 = x - 2.5$

$+2.5 +2.5$

$-5.5 = x$

Think: What numbers are 8 units from 0?

Rewrite the equations as two cases.

Since 2.5 is subtracted from $x$ add 2.5 to both sides of each equation.

The solution set is $\{10.5, -5.5\}$. 
Solutions to Absolute Value Equations

- Not all absolute-value equations have two solutions.
- If the absolute-value expression equals 0, there is one solution.
- If an equation states that an absolute-value is negative, there are no solutions.
Example: Special Case Absolute Value Equations

Solve the equation.

\[-8 = |x + 2| - 8\]

\[-8 = |x + 2| - 8\]
\[\underline{+8} \quad \underline{+ 8}\]
\[0 = |x + 2|\]

Since 8 is subtracted from \(|x + 2|\), add 8 to both sides to undo the subtraction.

There is only one case. Since 2 is added to x, subtract 2 from both sides to undo the addition.

\[0 = x + 2\]
\[\underline{-2} \quad \underline{-2}\]
\[-2 = x\]

The solution set is \{-2\}. 
Example: Special Case Absolute Value Equations

Solve the equation.

\[ 3 + |x + 4| = 0 \]

\[ 3 + |x + 4| = 0 \]
\[ -3 \]
\[ |x + 4| = -3 \]

Since 3 is added to \(|x + 4|\), subtract 3 from both sides to undo the addition.

Absolute value cannot be negative.

This equation has no solution. The solution set is the empty set, \(\emptyset\).
Remember!

Absolute value must be nonnegative because it represents a distance.
Your Turn:

Solve the equation.

\[ 2 - |2x - 5| = 7 \]

Since 2 is added to \(-|2x - 5|\), subtract 2 from both sides to undo the addition.

\[ 2 - |2x - 5| = 7 \]

\[ -2 |2x - 5| = 5 \]

Since \(|2x - 5|\) is multiplied by negative 1, divide both sides by negative 1.

\[ \frac{-|2x - 5|}{-1} = \frac{5}{-1} \]

Absolute value cannot be negative.

\[ |2x - 5| = -5 \]

This equation has no solution. The solution set is the empty set, \( \emptyset \).
Your Turn:

Solve the equation.

\[-6 + |x - 4| = -6\]

\[-6 + |x - 4| = -6\]
\[+6 \quad +6\]
\[|x - 4| = 0\]
\[x - 4 = 0\]
\[+4 \quad +4\]
\[x = 4\]

Since \(-6\) is added to \(|x - 4|\), add 6 to both sides. There is only one case. Since 4 is subtracted from \(x\), add 4 to both sides to undo the addition.
Example: Application

A support beam for a building must be 3.5 meters long. It is acceptable for the beam to differ from the ideal length by 3 millimeters. Write and solve an absolute-value equation to find the minimum and maximum acceptable lengths for the beam.

First convert millimeters to meters.

\[ 3 \, \text{mm} = 0.003 \, \text{m} \]

Move the decimal point 3 places to the left.

The length of the beam can vary by 0.003m, so find two numbers that are 0.003 units away from 3.5 on a number line.
Example: Continued

You can find these numbers by using the absolute-value equation $|x - 3.5| = 0.003$. Solve the equation by rewriting it as two cases.

**Case 1**

$x - 3.5 = 0.003$

$+3.5 = +3.5$

$x = 3.503$

**Case 2**

$x - 3.5 = -0.003$

$+3.5 = +3.5$

$x = 3.497$

The minimum length of the beam is 3.497 meters and the maximum length is 3.503 meters.
Your Turn:

Sydney Harbour Bridge is 134 meters tall. The height of the bridge can rise or fall by 180 millimeters because of changes in temperature. Write and solve an absolute-value equation to find the minimum and maximum heights of the bridge.

First convert millimeters to meters.

\[ 180 \text{ mm} = 0.180 \text{ m} \]

The height of the bridge can vary by 0.18 m, so find two numbers that are 0.18 units away from 134 on a number line.
Case 1
\[ x - 134 = 0.18 \]
\[ +134 = +134 \]
\[ x = 134.18 \]

Case 2
\[ x - 134 = -0.18 \]
\[ +134 = +134 \]
\[ x = 133.82 \]

The minimum height of the bridge is 133.82 meters and the maximum height is 134.18 meters.

Since 134 is subtracted from \( x \) add 134 to both sides of each equation.
SUMMARY
Absolute Value Equations

Equations Involving Absolute Value
If \( a \) is a positive real number and if \( u \) is any algebraic expression, then

\[
|u| = a \quad \text{is equivalent to} \quad u = a \quad \text{or} \quad u = -a.
\]

Note: If \( a = 0 \), the equation \(|u| = 0\) is equivalent to \( u = 0 \). If \( a < 0 \), the equation \(|u| = a\) has no real solution.
Absolute Value Equations

Steps for Solving Absolute Value Equations with One Absolute Value

Step 1: Isolate the expression containing the absolute value.
Step 2: Rewrite the absolute value equation as two equations: $u = a$ and $u = -a$, where $u$ is the algebraic expression in the absolute value symbol.
Step 3: Solve each equation.
Step 4: Verify your solution.
Joke Time

• What do you call cheese that doesn't belong to you?
  • Nacho cheese.

• Why do farts smell?
  • So the deaf can enjoy them too.

• What has four legs and one arm?
  • A happy pit bull.