Solving Proportions

Section 2-7
Goals

Goal

• To solve and apply proportions.
Vocabulary

- Proportion
- Cross Products
- Cross Products Property
What’s a Proportion?

• A *proportion* is an equation stating that two ratios are equal.

• Example:

\[
\frac{a}{b} = \frac{c}{d}
\]

\[b \neq 0 \quad \& \quad d \neq 0\]
Proportions

- If the ratio of \(a/b\) is equal to the ratio \(c/d\); then the following proportion can be written:

\[
\frac{a}{b} = \frac{c}{d}
\]

- The values \(a\) and \(d\) are the **extremes**. The values \(b\) and \(c\) are the **means**. When the proportion is written as \(a:b = c:d\), the extremes are in the first and last positions. The means are in the two middle positions.
In the proportion \( \frac{a}{b} = \frac{c}{d} \), the products \( a \cdot d \) and \( b \cdot c \) are called \textit{cross products}. You can solve a proportion for a missing value by using the Cross Products property.

**Cross Products Property**

<table>
<thead>
<tr>
<th>WORDS</th>
<th>NUMBERS</th>
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<tbody>
<tr>
<td>In a proportion, cross products are equal.</td>
<td>( \frac{2}{3} \times \frac{4}{6} = 2 \times 6 = 3 \times 4 )</td>
<td>( \text{If } \frac{a}{b} \text{ and } b \neq 0 ) and ( d \neq 0 ) then ( ad = bc ).</td>
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Cross Product Property

The product of the extremes equals the product of the means.

If \( \frac{a}{b} = \frac{c}{d} \)

then \( ad = bc \).
Solving a Proportion

To solve a proportion involving a variable, simply set the two cross products equal to each other. Then solve!

$$\frac{15 \cdot 25}{15} = \frac{275 \cdot x}{x}$$

$$375 = 275x$$

$$x = \frac{375}{275} = \frac{15}{11}$$
Example: Solving Proportions

Solve each proportion.

A. \( \frac{3}{9} = \frac{5}{m} \)
   \[\frac{3 \times 5}{9} \neq \frac{15}{m}\]
   \[3(m) = 5(9)\]
   \[3m = 45\]
   \[\frac{3m}{3} = \frac{45}{3}\]
   \[m = 15\]

B. \( \frac{6}{y - 3} = \frac{2}{7} \)
   \[\frac{6 \times 2}{y - 3} \neq \frac{12}{7}\]
   \[6(7) = 2(y - 3)\]
   \[42 = 2y - 6\]
   \[+6 +6\]
   \[48 = 2y\]
   \[\frac{48}{2} = \frac{2y}{2}\]
   \[24 = y\]
Your Turn:

Solve each proportion.

A. \( \frac{-5}{2} = \frac{y}{8} \)

\[\frac{-5}{2} \cdot 8 = y \cdot 2\]
\[2(y) = -5(8)\]
\[2y = -40\]
\[\frac{2y}{2} = \frac{-40}{2}\]
\[y = -20\]

Use cross products.

Divide both sides by 2.

B. \( \frac{g + 3}{5} = \frac{7}{4} \)

\[\frac{g + 3}{5} \cdot 4 = \frac{7}{4} \cdot 4\]
\[4(g + 3) = 5(7)\]
\[4g + 12 = 35\]
\[4g = 23\]
\[\frac{4g}{4} = \frac{23}{4}\]
\[g = 5.75\]

Use cross products.

Subtract 12 from both sides.

Divide both sides by 4.
Example: Solving Multi-Step Proportions

Solve the proportion \( \frac{b - 8}{5} = \frac{b + 3}{4} \).

\[
\begin{align*}
\frac{b - 8}{5} &= \frac{b + 3}{4} \\
5(4(b - 8)) &= 4(5(b + 3)) \\
4b - 32 &= 5b + 15 \\
4b - 4b &= 5b - 4b + 15 \\
-32 &= b + 15 \\
-32 - 15 &= b + 15 - 15 \\
-47 &= b
\end{align*}
\]

The equation \( 1 + \frac{b - 8}{5} = \frac{b + 3}{4} \) looks a lot like this example. Can you use cross products to find the value of \( b \)?

No, there are 2 terms on the left side of the equation.
Your Turn:

Solve the proportion $\frac{x - 2}{3} = \frac{x + 3}{5}$.

\[ \frac{x - 2}{3} = \frac{x + 3}{5} \]

\[ 5(x - 2) = 3(x + 3) \]

\[ 5x - 10 = 3x + 9 \]

\[ 5x - 3x - 10 = 3x - 3x + 9 \]

\[ 2x - 10 = 9 \]

\[ 2x - 10 + 10 = 9 + 10 \]

\[ 2x = 19 \]

\[ 2x = 19 \]

\[ 2x = \frac{19}{2} \]

\[ x = \frac{19}{2} \]
Example: Application of Ratios

The ratio of the number of bones in a human’s ears to the number of bones in the skull is 3:11. There are 22 bones in the skull. How many bones are in the ears?

\[
\frac{\text{ears}}{\text{skull}} \rightarrow \frac{3}{11}
\]

Write a ratio comparing bones in ears to bones in skull.

\[
\frac{3}{11} = \frac{x}{22}
\]

Write a proportion. Let \( x \) be the number of bones in ears.

\[
22\left(\frac{x}{22}\right) = 22\left(\frac{3}{11}\right)
\]

Since \( x \) is divided by 22, multiply both sides of the equation by 22.

\[
x = 6
\]

There are 6 bones in the ears.
The ratio of red marbles to green marbles is 6:5. There are 18 red marbles. How many green marbles are there?

Write a ratio comparing green to red marbles.

\[ \frac{5}{6} = \frac{x}{18} \]

Write a proportion. Let \( x \) be the number of green marbles.

Since \( x \) is divided by 18, multiply both sides by 18.

\[ 18 \left( \frac{5}{6} \right) = 18 \left( \frac{x}{18} \right) \]

There are 15 green marbles.
Joke Time

• What do prisoners use to call each other?
  • Cell phones.

• What do you get when you cross a snowman with a vampire?
  • Frostbite.

• What lies at the bottom of the ocean and twitches?
  • A nervous wreck.