Variables and Expressions

Section 1-1
Goals

Goal

• To write algebraic expressions.
Vocabulary

- Quantity
- Variable
- Constant
- Algebraic expression
- Term
- Factor
- Numerical expression
Definition

• **Quantity** – A mathematical *quantity* is anything that can be measured or counted.
  – How much there is of something.
  – A single group, generally represented in an expression using parenthesis () or brackets [ ].

• Examples:
  – numbers, number systems, volume, mass, length, people, apples, chairs.
  – \((2x + 3), (3 - n), [2 + 5y]\).
Definition

• **Variable** – anything that can vary or change in value.
  – In algebra, \( x \) is often used to denote a variable.
  – Other letters, generally letters at the end of the alphabet (\( p, q, r, s, t, u, v, w, x, y, \) and \( z \)) are used to represent variables.
  – A variable is “just a number” that can change in value.

• **Examples:**
  – A child’s height
  – Outdoor temperature
  – The price of gold
Definition

• *Constant* – anything that does not vary or change in value (a number).
  – In algebra, the numbers from arithmetic are constants.
  – Generally, letters at the beginning of the alphabet (a, b, c, d) used to represent constants.

• Examples:
  – The speed of light
  – The number of minutes in a hour
  – The number of cents in a dollar
  – \( \pi \).
Definition

- \textit{Algebraic Expression} – a mathematical phrase that may contain variables, constants, and/or operations.
- Examples: $5x + 3$, $y/2 - 4$, $xy - 2x + y$, $3(2x + 7)$, $\frac{2a+b}{5c}$
Definition

• **Term** – any number that is added or subtracted.
  – In the algebraic expression $x + y$, $x$ and $y$ are terms.

• Example:
  – The expression $x + y - 7$ has 3 terms, $x$, $y$, and $7$. $x$ and $y$ are variable terms; their values vary as $x$ and $y$ vary. $7$ is a constant term; $7$ is always $7$. 
Definition

- **Factor** – any number that is multiplied.
  - In the algebraic expression $3x$, $x$ and 3 are factors.

- **Example:**
  - $5xy$ has three factors; 5 is a constant factor, $x$ and $y$ are variable factors.
Example: Terms and Factors

• The algebraic expression $5x + 3$;
  – has two terms $5x$ and $3$.
  – the term $5x$ has two factors, $5$ and $x$. 
Definition

• *Numerical Expression* – a mathematical phrase that contains only constants and/or operations.

• Examples: $2 + 3$, $5 \cdot 3 - 4$, $4 + 20 - 7$, $(2 + 3) - 7$, $(6 \times 2) \div 20$, $5 \div (20 \times 3)$
Multiplication Notation

In expressions, there are many different ways to write multiplication.

1) \( ab \)
2) \( a \cdot b \)
3) \( a(b) \) or \( (a)b \)
4) \( (a)(b) \)
5) \( a \times b \)

We are not going to use the multiplication symbol (\( \times \)) anymore. Why?
Can be confused with the variable \( x \).
Division Notation

Division, on the other hand, is written as:

1) \( \frac{x}{3} \)

2) \( x \div 3 \)

In algebra, normally write division as a fraction.
Translate Words into Expressions

• To Translate word phrases into algebraic expressions, look for words that describe mathematical operations (addition, subtraction, multiplication, division).
What words indicate a particular operation?

Addition
- Sum
- Plus
- More than
- Increase(d) by
- Perimeter
- Deposit
- Gain
- Greater (than)
- Total

Subtraction
- Minus
- Take away
- Difference
- Reduce(d) by
- Decrease(d) by
- Withdrawal
- Less than
- Fewer (than)
- Loss of
# Words for Operations - Examples

<table>
<thead>
<tr>
<th>Addition</th>
<th>Mathematical Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>plus</td>
<td>“a number plus 2”</td>
</tr>
<tr>
<td>and</td>
<td>“3 and a number”</td>
</tr>
<tr>
<td>added to</td>
<td>“8 added to a number”</td>
</tr>
<tr>
<td>greater than</td>
<td>“3 greater than a number”</td>
</tr>
<tr>
<td>more than</td>
<td>“3 more than a number”</td>
</tr>
<tr>
<td>increased by</td>
<td>“a number increased by 2”</td>
</tr>
<tr>
<td>total</td>
<td>“the total length”</td>
</tr>
<tr>
<td>sum of</td>
<td>“The sum of the length and width”</td>
</tr>
<tr>
<td></td>
<td>$x + 2$</td>
</tr>
<tr>
<td></td>
<td>$3 + n$</td>
</tr>
<tr>
<td></td>
<td>$x + 8$</td>
</tr>
<tr>
<td></td>
<td>$n + 3$</td>
</tr>
<tr>
<td></td>
<td>$y + 3$</td>
</tr>
<tr>
<td></td>
<td>$y + 2$</td>
</tr>
<tr>
<td></td>
<td>$l_1 + l_2 + ...$</td>
</tr>
<tr>
<td></td>
<td>$l + w$</td>
</tr>
</tbody>
</table>
## Subtraction

<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
<th>Algebraic Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>minus</td>
<td>“a number minus 2”</td>
<td>( x - 2 )</td>
</tr>
<tr>
<td>difference between</td>
<td>“the difference between a number and 8”</td>
<td>( x - 8 )</td>
</tr>
<tr>
<td>from</td>
<td>“2 from a number”</td>
<td>( n - 2 )</td>
</tr>
<tr>
<td>less</td>
<td>“a number less 3”</td>
<td>( n - 3 )</td>
</tr>
<tr>
<td>less than</td>
<td>“3 less than a number”</td>
<td>( y - 3 )</td>
</tr>
<tr>
<td>fewer than</td>
<td>“2 fewer than a number”</td>
<td>( y - 2 )</td>
</tr>
<tr>
<td>decreased by</td>
<td>“a number decreased by 2”</td>
<td>( x - 2 )</td>
</tr>
<tr>
<td>take away</td>
<td>“a number take away 2”</td>
<td>( x - 2 )</td>
</tr>
</tbody>
</table>
What words indicate a particular operation?

**Multiply**
- Times
- Product
- Multiplied by
- Of
- Twice ($\times 2$), double ($\times 2$), triple ($\times 3$), etc.
- Half ($\times \frac{1}{2}$), Third ($\times \frac{1}{3}$), Quarter ($\times \frac{1}{4}$)
- Percent (of)

**Divide**
- Quotient
- Divided by
- Half ($\div 2$), Third ($\div 3$), Quarter ($\div 4$)
- Into
- Per
- Percent (out of 100)
- Split into ___ parts
Words for Operations - Examples

<table>
<thead>
<tr>
<th>Multiplication</th>
<th>Example</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>times</td>
<td>“5 times a number”</td>
<td>$5n$</td>
</tr>
<tr>
<td>product</td>
<td>“The product of 3 and a number”</td>
<td>$3y$</td>
</tr>
<tr>
<td>at</td>
<td>“3 at 1.59”</td>
<td>$3 \cdot 1.59$</td>
</tr>
<tr>
<td>double, triple, etc.</td>
<td>“double a number”</td>
<td>$2x$</td>
</tr>
<tr>
<td>twice</td>
<td>“twice a number”</td>
<td>$2y$</td>
</tr>
<tr>
<td>of (fractions of)</td>
<td>“three-fourths of a number”</td>
<td>$\left(\frac{3}{4}\right)y$</td>
</tr>
</tbody>
</table>
### Words for Operations - Examples

**Division**

<table>
<thead>
<tr>
<th>Description</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>quotient of</td>
<td>“The quotient of 5 and a number” $\frac{5}{n}$ or $5 \div n$</td>
</tr>
<tr>
<td>Half of</td>
<td>“half of a number” $\frac{n}{2}$</td>
</tr>
<tr>
<td>goes into</td>
<td>“a number goes into 6 twice” $\frac{6}{n} = 2$</td>
</tr>
<tr>
<td>per</td>
<td>“The price is $8 per 50” $P = \frac{8}{50}$</td>
</tr>
</tbody>
</table>
Writing an Algebraic Expression for a Verbal Phrase.

Writing an algebraic expression with addition.

Two plus a number n

2 + n

Order of the wording Matters
Writing an Algebraic Expression for a Verbal Phrase.

Writing an algebraic expression with addition.

Two more than a number

\[ x + 2 \]

Order of the wording Matters
Writing an Algebraic Expression for a Verbal Phrase.

Writing an algebraic expression with subtraction.

The difference of seven and a number \( n \)

\[
7 - n
\]

Order of the wording Matters
Writing an Algebraic Expression for a Verbal Phrase.

Writing an algebraic expression with subtraction.

Eight less than a number

\[ y - 8 \]

Order of the wording Matters
Writing an Algebraic Expression for a Verbal Phrase.

Writing an algebraic expression with multiplication.

\[ \frac{1}{3} \cdot n \]

one-third of a number \( n \).

Order of the wording Matters
Writing an Algebraic Expression for a Verbal Phrase.

Writing an algebraic expression with division.

The quotient of a number $n$ and 3

\[
\frac{n}{3}
\]
Example

“Translating” a phrase into an algebraic expression:

Nine more than a number $y$

Can you identify the operation?

“more than” means add

Answer: $y + 9$
Example

“Translating” a phrase into an algebraic expression:

4 less than a number $n$

Identify the operation?

“less than” means add

Determine the order of the variables and constants.

Answer: $n - 4$.

Why not $4 - n$??????
Example

“Translating” a phrase into an algebraic expression:

A quotient of a number $x$ and 12

Can you identify the operation?

“quotient” means divide

Determine the order of the variables and constants.

Answer: $\frac{x}{12}$.

Why not $\frac{12}{x}$??
Example

“Translating” a phrase into an algebraic expression, this one is harder……

5 times the quantity 4 plus a number c

Can you identify the operation(s)?

“times” means multiple and “plus” means add

What does the word quantity mean?

that “4 plus a number c” is grouped using parenthesis

Answer: $5(4 + c)$
Your turn:

1) $m$ increased by 5. \hspace{1cm} 1) $m + 5$
2) 7 times the product of $x$ and $t$. \hspace{1cm} 2) $7xt$
3) 11 less than 4 times a number. \hspace{1cm} 3) $4n - 11$
4) two more than 6 times a number. \hspace{1cm} 4) $6n + 2$
5) the quotient of a number and 12. \hspace{1cm} 5) $\frac{x}{12}$
Your Turn:

Which of the following expressions represents 7 times a number decreased by 13?

a. 7x + 13
b. 13 - 7x
c. 13 + 7x

✔ d. 7x - 13
Your Turn:

Which one of the following expressions represents 28 less than three times a number?

1. 28 - 3x

2. 3x - 28

3. 28 + 3x

4. 3x + 28
Your Turn:

Match the verbal phrase and the expression

1. Twice the sum of x and three  
   D. \(2(x + 3)\)

2. Two less than the product of 3 and x  
   E. \(3(x - 2)\)

3. Three times the difference of x and two  
   B. \(3x + 2\)

4. Three less than twice a number x  
   A. \(2x - 3\)

5. Two more than three times a number x  
   C. \(3x - 2\)
Translate an Algebraic Expression into Words

• We can also start with an algebraic expression and then translate it into a word phrase using the same techniques, but in reverse.

• Is there only one way to write a given algebraic expression in words?
  – No, because the operations in the expression can be described by several different words and phrases.
Example: Translating from Algebra to Words

Give two ways to write each algebra expression in words.

A. $9 + r$
   - the sum of 9 and $r$
   - 9 increased by $r$

B. $q - 3$
   - the difference of $q$ and 3
   - 3 less than $q$

C. $7m$
   - the product of $m$ and 7
   - $m$ times 7

D. $j ÷ 6$
   - the quotient of $j$ and 6
   - $j$ divided by 6
Your Turn:

Give two ways to write each algebra expression in words.

a. 4 - n
   4 decreased by n
   n less than 4

b. \( \frac{t}{5} \)
   the quotient of t and 5
   t divided by 5

c. 9 + q
   the sum of 9 and q
   q added to 9

d. 3(h)
   the product of 3 and h
   3 times h
Your Turn:

Which of the following verbal expressions represents $2x + 9$?

1. 9 increased by twice a number
2. a number increased by nine
3. twice a number decreased by 9
4. 9 less than twice a number

✓ 1. 9 increased by twice a number
Your Turn:

Which of the following expressions represents the sum of 16 and five times a number?

1. $5x - 16$
2. $16x + 5$
3. $16 + 5x$  \(\checkmark\)
4. $16 - 5x$
Your Turn:

CHALLENGE
Write a verbal phrase that describes the expression

- $4(x + 5) - 2$
  - Four times the sum of $x$ and 5 minus two
- $7 - 2(x ÷ 3)$
  - Seven minus twice the quotient of $x$ and three
- $m ÷ 9 - 4$
  - The quotient of $m$ and nine, minus four
Your Turn:

Define a variable to represent the unknown and write the phrase as an expression.

- **Six miles more than yesterday**
  - Let \( x \) be the number of miles for yesterday
  - \( x + 6 \)

- **Three runs fewer than the other team scored**
  - Let \( x \) = the amount of runs the other team scored
  - \( x - 3 \)

- **Two years younger than twice the age of your cousin**
  - Let \( x \) = the age of your cousin
  - \( 2x - 2 \)
Patterns

Mathematicians ...

• look for patterns
• find patterns in physical or pictorial models
• look for ways to create different models for patterns
• use mathematical models to solve problems
# Number Patterns

<table>
<thead>
<tr>
<th>Term Number</th>
<th>Term</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1(2)</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2(2)</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>3(2)</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>4(2)</td>
</tr>
<tr>
<td>( n )?</td>
<td></td>
<td>( n(2) )</td>
</tr>
</tbody>
</table>
# Number Patterns

<table>
<thead>
<tr>
<th>Term Number</th>
<th>Term</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>19</td>
<td>3(5) + 4</td>
</tr>
<tr>
<td>2</td>
<td>24</td>
<td>4(5) + 4</td>
</tr>
<tr>
<td>3</td>
<td>29</td>
<td>5(5) + 4</td>
</tr>
<tr>
<td>4</td>
<td>34</td>
<td>6(5) + 4</td>
</tr>
</tbody>
</table>

**What’s the same?**

1. 19
2. 24
3. 29
4. 34

**What’s different?**

1. 3(5) + 4
2. 4(5) + 4
3. 5(5) + 4
4. 6(5) + 4

**How does the different part relate to the term number?**

(n + 2)(5) + 4
## Number Patterns

<table>
<thead>
<tr>
<th>Term Number</th>
<th>Term</th>
<th>Expression</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>$3 - 2(0)$</td>
<td>same?</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>$3 - 2(1)$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
<td>$3 - 2(2)$</td>
<td>different part relate to the term number?</td>
</tr>
<tr>
<td>4</td>
<td>-3</td>
<td>$3 - 2(3)$</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td></td>
<td>$3 - 2(n - 1)$</td>
<td></td>
</tr>
</tbody>
</table>
Writing a Rule to Describe a Pattern

• Now let's try a real-life problem.
Bonjour! My name is Fernando. I am preparing to cook a GIGANTIC home-cooked Italian meal for my family. The only problem is I don’t know yet how many people are coming. The more people that come, the more spaghetti I will need to buy.

**Shopping List**

- Guests = ?
- Bags of Spaghetti = ?
From all the meals I have cooked before I know:

For 1 guest I will need 2 bags of spaghetti,
For 2 guests I will need 5 bags of spaghetti,
For 3 guests I will need 8 bags of spaghetti,
For 4 guests I will need 11 bags of spaghetti.
Here is the table of how many bags of spaghetti I will need to buy:

<table>
<thead>
<tr>
<th>Number of Guests</th>
<th>Bags of Spaghetti</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
</tr>
</tbody>
</table>
The numbers in the ‘spaghetti’ column make a pattern:

2 \rightarrow 5 \rightarrow 8 \rightarrow 11

+ 3 \rightarrow + 3 \rightarrow + 3

What do we need to add on each time to get to the next number?
We say there is a **COMMON DIFFERENCE** between the numbers.

We need to add on the same number every time.

What is the common difference for this sequence? 3
Now we know the common difference we can start to work out the **MATHEMATICAL RULE**.

The mathematical rule is the algebraic expression that lets us find any value in our pattern.
We can use our common difference to help us find the mathematical rule.

We always multiply the common difference by the **TERM NUMBER** to give us the first step of our mathematical rule.

What are the term numbers in my case are?

**NUMBER OF GUESTS**
So if we know that step one of finding the mathematical rule is:

Common Difference $\times$ Term Numbers

then what calculations will we do in this example?

Common Difference $\times$ Term Numbers

$3 \times$ Number of Guests
<table>
<thead>
<tr>
<th>Number of Guests (n)</th>
<th>Bags of Spaghetti</th>
<th>3n</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>12</td>
</tr>
</tbody>
</table>

We will add a column to our original table to do these calculations:
We are trying to find a mathematical rule that will take us from:

Number of Guests

↓

Number of Bags of Spaghetti

At the moment we have:

$3n$

Does this get us the answer we want?
3n gives us: Bags of Spaghetti

| 3  | 6  | 9  | 12 |
|----|----|----|----|---|
| -1 | -1 | -1 | -1 |→ 2 | 5 | 8 | 11 |

What is the difference between all the numbers on the left and all the numbers on the right?

-1
<table>
<thead>
<tr>
<th>Number of Guests (n)</th>
<th>Bags of Spaghetti</th>
<th>3n</th>
<th>3n - 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
<td>12</td>
<td>11</td>
</tr>
</tbody>
</table>

We will now add another column to our table to do these calculations:
Does this new column get us to where we are trying to go?

So now we know our mathematical rule:

$$3n - 1$$
Your Turn:

- The table shows how the cost of renting a scooter depends on how long the scooter is rented. What is a rule for the total cost? Give the rule in words and as an algebraic expression.

<table>
<thead>
<tr>
<th>Hours</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$17.50</td>
</tr>
<tr>
<td>2</td>
<td>$25.00</td>
</tr>
<tr>
<td>3</td>
<td>$32.50</td>
</tr>
<tr>
<td>4</td>
<td>$40.00</td>
</tr>
<tr>
<td>5</td>
<td>$47.50</td>
</tr>
</tbody>
</table>

Answer:
Multiply the number of hours by 7.5 and add 10. $7.5n + 10$
Joke Time

• Where do you find a dog with no legs?
  • Right where you left him.

• What do you call a cow with no legs?
  • Ground beef.

• What do you call a cow with 2 legs?
  • Lean beef.